

BC Review FINAL, NO Calculator Permitted (unless stated otherwise)
Do all work on separate notebook paper

(a) $\int x \sin(2x) dx =$

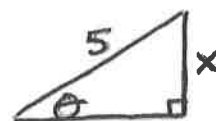
(b) $\int \frac{dx}{x^2 - 6x + 8} =$

2. Write an integral expression which gives the area of the region inside the polar curve $r = 4 \cos \theta$ and outside $r = 2$.

3. Given $\frac{dy}{dx} = \frac{xy}{2}$. Let $f(x)$ be the particular solution to the given differential equation with initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$.

4. If $3xy + 2x = 1 - 2y$, then when $x = -1$, $\frac{dy}{dx} = ?$

5. If in the triangle at right, θ decreases by 4 rad/min, at what rate is x changing in units/min when $x = 4$?



6. Write an integral equation which gives the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$ for $0 \leq t \leq \frac{\pi}{2}$.

7. If f is a function such that $f'(x) = \sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is?

8. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y = ?$

9. (a) $\int \tan^2 x dx =$

(b) $\int \sin^2 x dx =$

10. $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2}\right)}{h} =$

11. The coefficient of x^3 in the Taylor series for e^{2x} about $x = 0$ is?

12. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converge?

13. Which of the following converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$

II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

15. The population $P(t)$ of unicorns in a forest satisfies the logistic differential equation $\frac{dP}{dt} = 3P - \frac{P^2}{6000}$.
- (a) If $P(0) = 4000$, what is $\lim_{t \rightarrow \infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.
- (b) If $P(0) = 10,000$, what is $\lim_{t \rightarrow \infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.
- (c) If $P(0) = 20,000$, what is $\lim_{t \rightarrow \infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.
- (d) If $P(0) = 4000$, what is the population when it is growing the fastest? Where is the solution curve concave up? Concave down? Justify your answer.

16. (Calculator Permitted) Given $\frac{dx}{dt} = \cos(t^3)$ and $\frac{dy}{dt} = 3\sin(t^2)$ for $0 \leq t \leq 3$. At time $t = 2$, the object at position $(4, 5)$.
- (a) Find the speed of the object at time $t = 2$.
- (b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 1$.
- (c) Find the position of the object at time $t = 3$.

17. (Calculator Permitted) Given $f(x) = \frac{1}{3} + \frac{2x}{9} + \frac{3x^2}{27} + \dots + \frac{(n+1)}{3^{n+1}}x^n + \dots$

(a) $\lim_{x \rightarrow 0} \frac{f(x) - \frac{1}{3}}{x}$.

- (b) Write the first three nonzero terms and the general term for the infinite series that represents $\int_0^1 f(x) dx$.
- (c) Find the sum of the series found in part (b).

18. (Calculator Permitted) The function f has derivatives of all orders for all real numbers x . Assume that $f(2) = 5$, $f'(2) = -3$, $f''(2) = 4$, $f'''(2) = -1$, and $|f^{(4)}(x)| \leq 3$ for all x in $[2, 2.2]$.
- (a) Write the third-degree Taylor polynomial for f about $x = 2$.
- (b) Use your answer to (a) to approximate $f(2.15)$. Give your answer correct to five decimal places.
- (c) Use the Lagrange error bound on the approximation of $f(2.15)$ to explain why $f(2.15) \neq 4.7$.